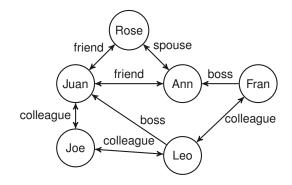
A Recursive Approach for Defining Reachability Queries over Graph Databases

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Graph Structured Data is now everywhere



- Facebook, Twitter
- DBPedia (Wikipedia represented as a graph)
- Biological databases, geological databases, social databases...

Web of Data needs graphs

Semantic Web:

Integrate semantic content to web resources



Semantic Web (some projects)







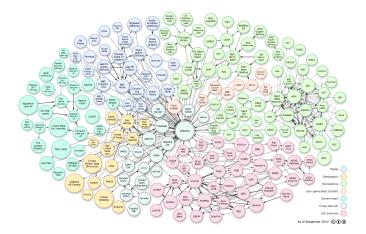


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Aerial View of Linked Data sources



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Semantic Web \longrightarrow Graph Database

Web resources are modeled as nodes of a graph, edges are relations between resources

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Semantic Web — Graph Database

Web resources are modeled as nodes of a graph, edges are relations between resources



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Querying Semantic Web:

SPARQL:

SPARQL Protocol and RDF Query Language

- Standardized query language for RDF
- Query language with strong relational flavor

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Querying Semantic Web:

SPARQL:

SPARQL Protocol and RDF Query Language

- Standardized query language for RDF
- Query language with strong relational flavor
- Think of Relational Algebra or SQL over graphs (selection, projection, joins, unions, etc...)

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Connectivity Queries: a novel challenge

Talk about paths in graphs



Connectivity Queries: a novel challenge

Talk about paths in graphs

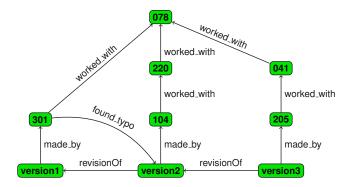
- Is node A connected to node B?
- Is it connected by a path satisfying certain properties?
- Other possibilities: return (simple) paths, aggregation, etc.

Connectivity Queries: a novel challenge

In a social network: Am I connected to a superstar?

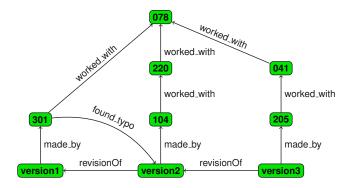
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- Workflows (biological): how is the path from process A to process B?
- Maps: shortest path form A to B?

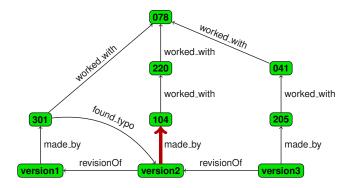


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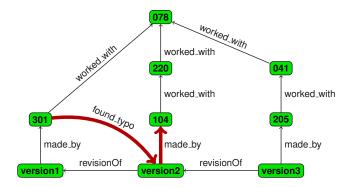
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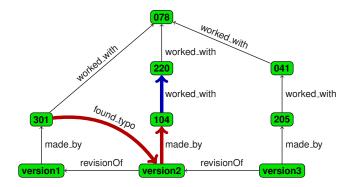
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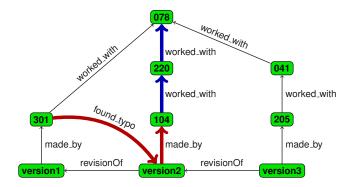
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This talk:

Connectivity queries for Semantic Web databases

Juan L. Reutter

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DataLab Dept. of Computer Science PUC Chile

Outline

- 1. Graph databases Most studied connectivity queries
- 2. Relationship between connectivity and recursion An algebra for querying graphs

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3. Where to go from here

Outline

- 1. Graph databases Most studied connectivity queries
- 2. Relationship between connectivity and recursion An algebra for querying graphs

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3. Where to go from here

Graph databases (for this talk):

We abstract them as triples:



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Graph databases (for this talk):

We abstract them as triples:



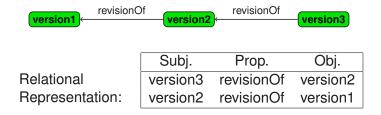
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A graph database is a collection of triples.

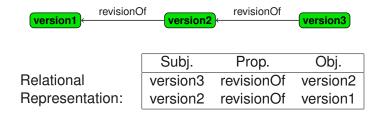
RDF graphs can be represented by relational databases.

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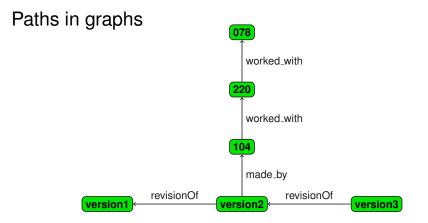


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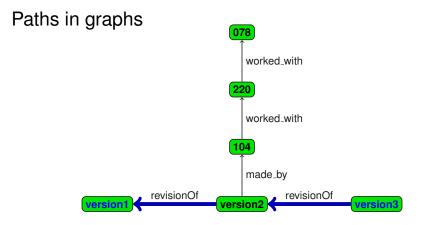
RDF graphs can be represented by relational databases.



The distinction is in the queries that we want to ask (such as connectivity queries)

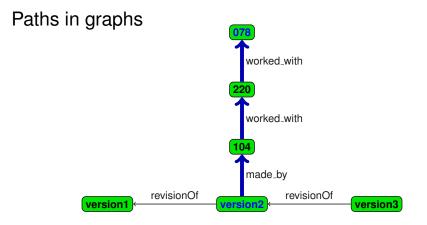


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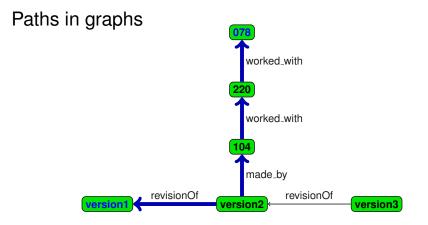
The label of the path is

revisionOf · revisionOf



The label of the path is

 $made_by \cdot worked_with \cdot worked_with$



The label of the path is

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Regular Path Queries (RPQs)

One of the most studied connectivity query languages

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- Part of SPARQL (query language for RDF)
- Most graph DB systems claim to support it

Regular Path Queries (RPQs)

- One of the most studied connectivity query languages
- Part of SPARQL (query language for RDF)
- Most graph DB systems claim to support it

Idea:

Check paths that are given by a certain regular expression

Regular Path Queries (RPQs), formally

Let Σ be a set of properties

RPQs are of the form

(x, e, y)

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Where e is a regular expression over Σ .

Example in Social Networks

 $(x, works_with^*, y),$

finds all people that are connected via co-worker relationship.

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Example in Crime Networks

(*x*, reports⁺, *Boss*)

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finds all people that reports directly or indirectly to the Boss.

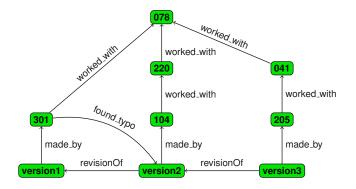
Example in Crime Networks

```
(x, reports<sup>+</sup>, Boss)
```

finds all people that reports directly or indirectly to the Boss.

```
(x, \Sigma^* \cdot \text{reports} \cdot \Sigma^* \cdot \text{reports} \cdot \Sigma^*, Boss)
```

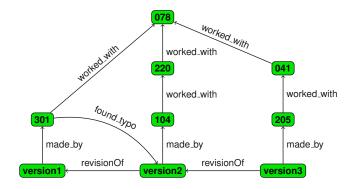
finds all people connected to Boss by a path that runs via at least two intermediaries.



Revision and people involved

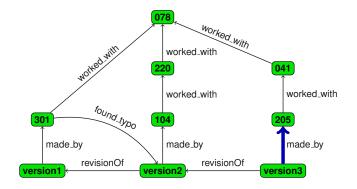
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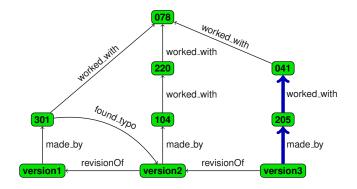
Revision and people involved

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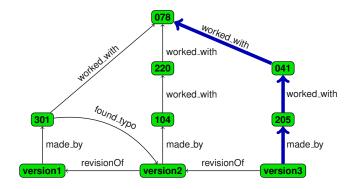
Revision and people involved

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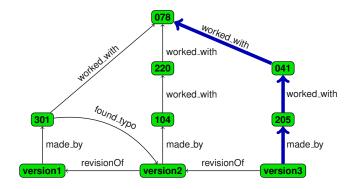
Revision and people involved

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Revision and people involved

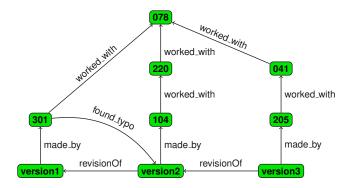
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RPQs is used as a primitive for querying paths

- Supported by various systems
- Simple, declarative language (easy to state what you want)

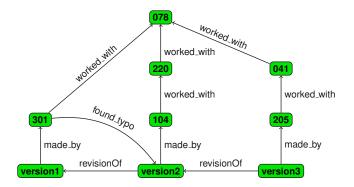
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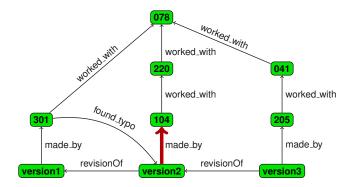
- Really efficient evaluation: Automata techniques allow to use fast reachability algorithms to evaluate RPQs.
- Lots of extensions

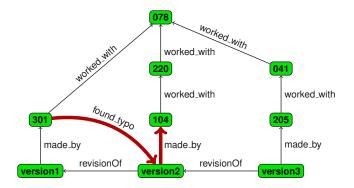


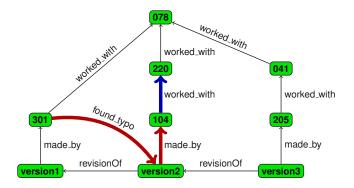
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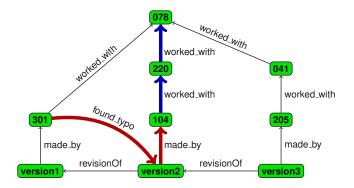
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 People who might have made a typo: [made_by⁻ · found_typo⁻] · (worked_with)* 104, 220, 078

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- 1. Graph databases Most studied connectivity queries
- 2. Relationship between connectivity and recursion An algebra for querying graphs

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3. Where to go from here

In practice, things are not that simple

Graph DB systems struggle to support RPQs.

Neo4j only supports concatenation, star

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In practice, things are not that simple

While RPQs are now part of SPARQL

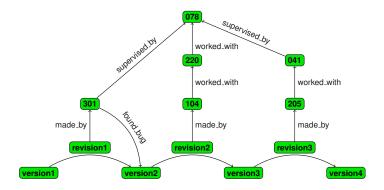
- Semantics not clearly defined (many changes in last years)
- no clear guidelines for implementation

Problem with RPQs: limited expressivity

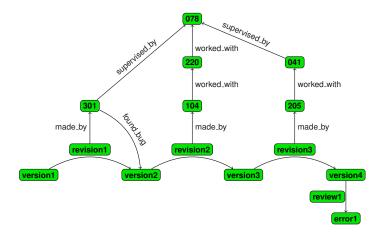
to study it we use SPARQL (with RPQs)

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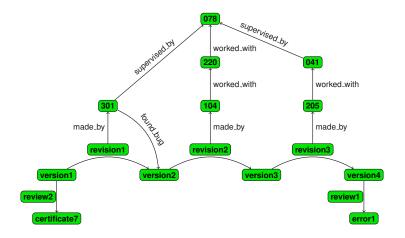
more or less like SQL + RPQs

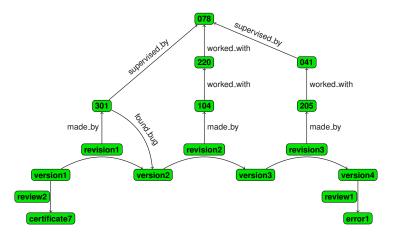


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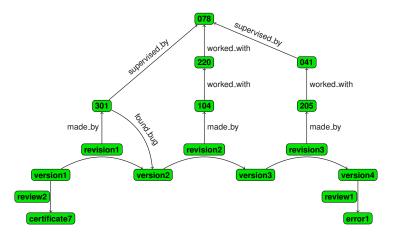
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Find all versions with an error

*Q*₁ : That originate from a valid version And return the latest revision



Find all versions with an error

Q₂: That originate from a valid version And the person responsible for this

What is the problem?

- Q1 can be expressed using SPARQL (essentially SQL + RPQs)
- Q₂ can NOT be expressed in SPARQL
- Conclusion:
 - Need to reason while moving along paths
 - Cant do this using RPQs or similar primitives

Problem with RPQs: implementation

Simple and efficient implementation using automata theory

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Problem with RPQs: implementation

Traditionally, DB systems do not implement techniques that rely on automata theory.

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XML and XPath

Why not base implementation on relational queries

Understanding connectivity queries from a relational point of view

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Understanding connectivity queries from a relational point of view

What are RPQs? What does (x, a^*, y) means?



► Let S ∘ S' be the composition of binary relations S and S':

 $\mathcal{S} \circ \mathcal{S}' = \{(x,z) \mid (x,y) \in \mathcal{S} \land (y,z) \in \mathcal{S}'\}$

What does (x, a^*, y) means?

Take binary relation A given by all x, y that are connected via label a in the graph.

 $(x, a^+, y) = A \cup A \circ A \cup A \circ A \circ A \cup \ldots$

Compose *A* with itself over and over again... until we reach a fixed point.

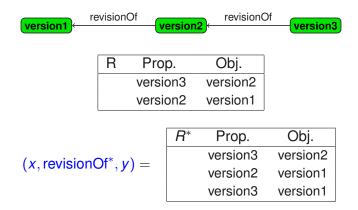
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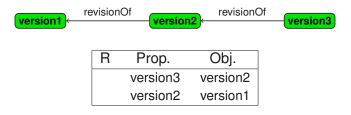


(x, revisionOf*, y)



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$(x, revisionOf^*, y)$



$(x, revisionOf^*, y) =$	<i>R</i> *	Prop.	Obj.
		version3	version2
		version2	version1
		version3	version1
		version3	version3
		version2	version2
		version1	version1

But from the eyes of relational algebra...

Composition is just a join!

$$S \circ S' = S \bigotimes_{2=1'}^{1,2'} S'$$

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Thus reachability is just recursive iteration of joins.

TriAL: an algebra for graphs

We now define an algebra for triples, that:

- can express RPQs
- can even express queries such as Q2

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Is based on relational algebra

Composing ternary relation

We need to manage triples... No obvious way to do it

 $(x,y,x) \circ (x',y',z')$

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We take the approach of relational algebra, and define all possible compositions.







$R \bowtie R'$

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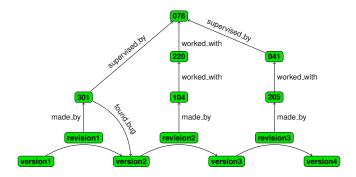
$R \stackrel{1,3',3}{\bowtie} R'$

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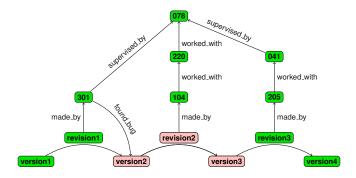


$R \overset{1,3',3}{\bowtie} R'_{2=1'}$

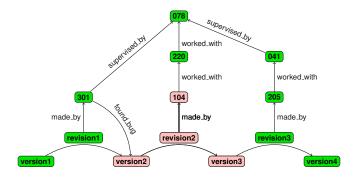
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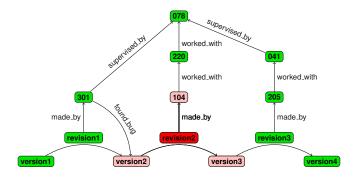
 $E \underset{2=1'}{\overset{1,3',3}{\bowtie}} E$



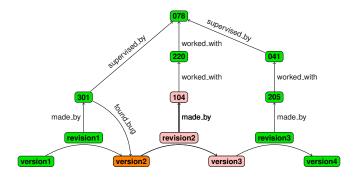
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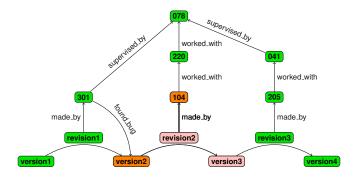
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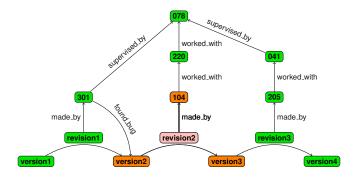
 $E_{2=1'}^{1,3',3}E$



 $E^{1,3',3}_{\substack{\boxtimes\\ 2=1'}}E$



 $E^{1,3',3}_{\underset{2=1'}{\bowtie}}E$



 $E \overset{1,3',3}{\bowtie}_{2=1'} E$

TriAL: An algebra of triples

R: set of triples
Relational representation of a an RDF graph

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A TriAL expression is built using

- Set R of triples
- ► Joins 🖂
- ► Union ∪
- ► Difference \

Adding recursion – TriAL*

For binary reachability we just iterate the join

- ► For triples things are not symmetric
 - In particular some joins are not associative

So we need both left and right star

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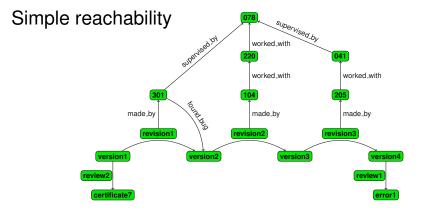
 $(e \bowtie)^* = \emptyset \cup e \cup e \bowtie e \cup (e \bowtie e) \bowtie e \cup \dots,$

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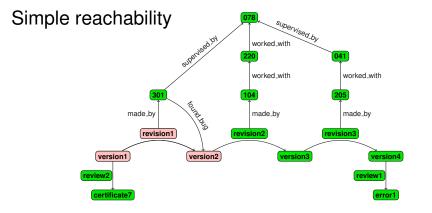
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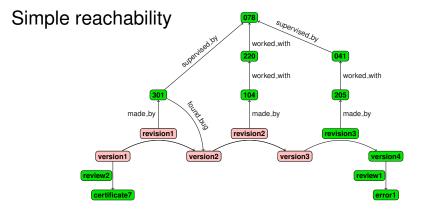
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 $(E \bowtie_{3=1'}^{1,2',3'})^*$



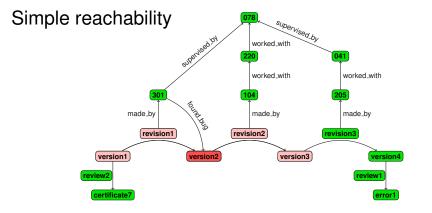
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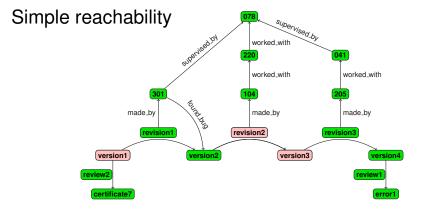
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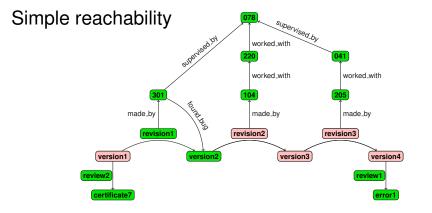
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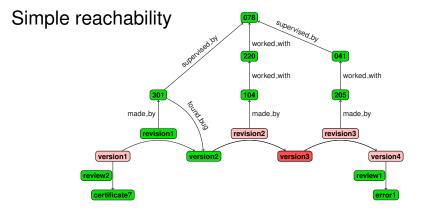
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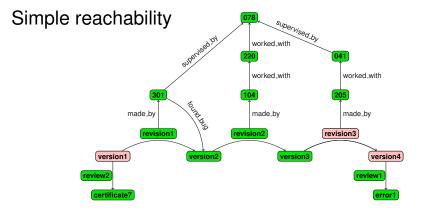
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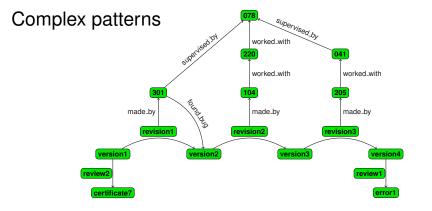
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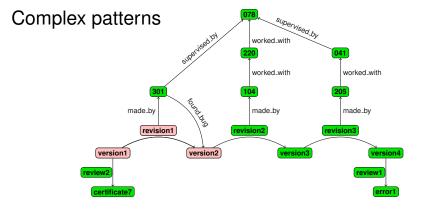
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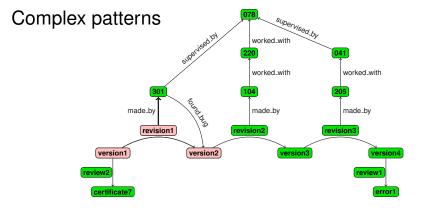
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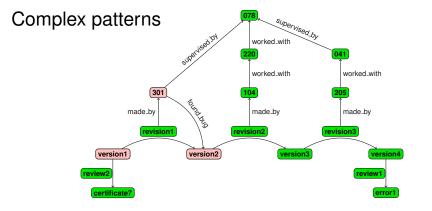
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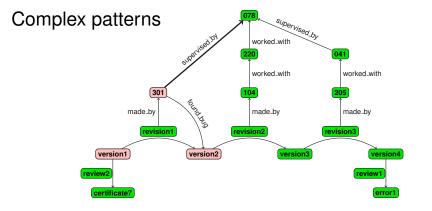
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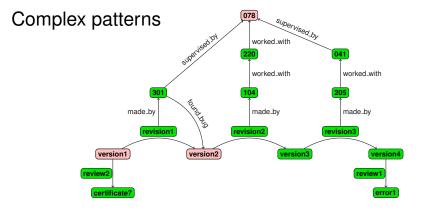
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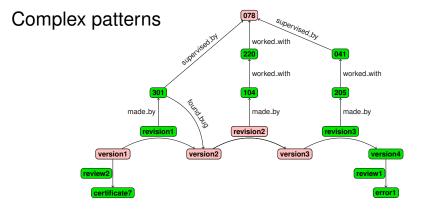
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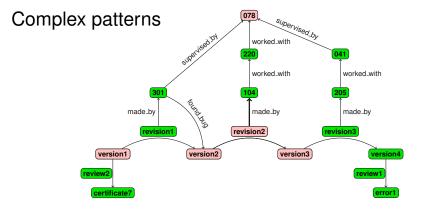
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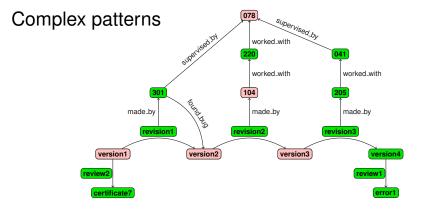
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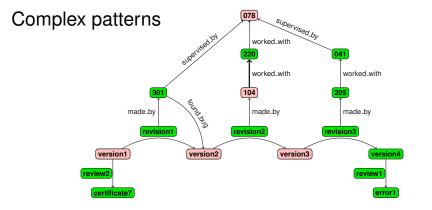
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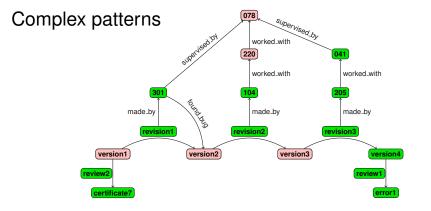
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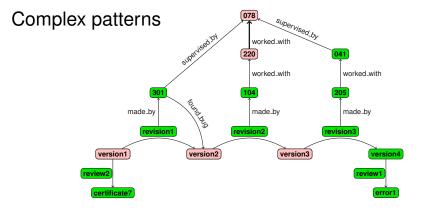
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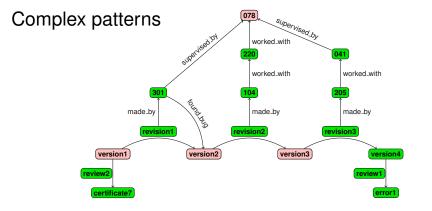
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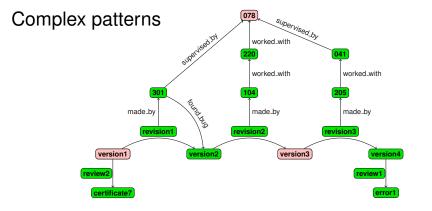
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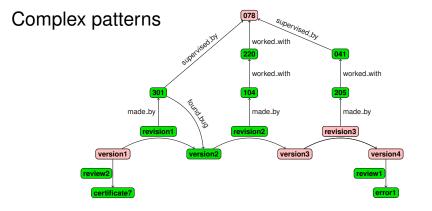
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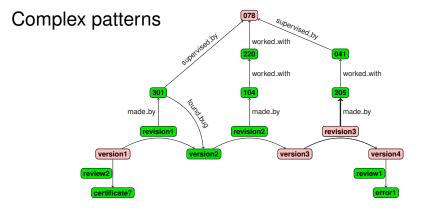
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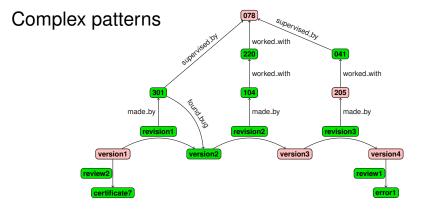
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$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



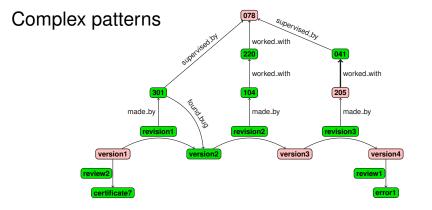
*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



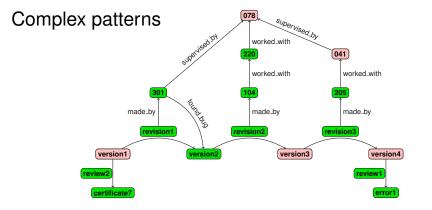
*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



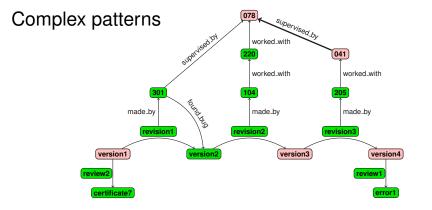
*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



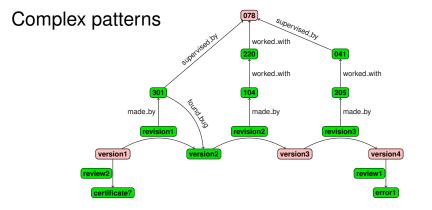
*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



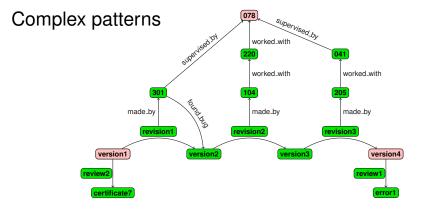
*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$



*Q*₂ : That originate from a valid version And the person responsible

$$((E \bigotimes_{2=1'}^{1,3',3})^* \bigotimes_{3=1',2=2'}^{1,2,3'})^*$$

- Similar complexity bounds to RPQs
- Evaluation algorithm uses dynamic programming

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TriAL*: fragment of relational algebra

- well known formalism
- can be translated into SQL- like statements (or datalog-like)
- fits right onto relational query implementations but we might need new heuristics

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Other theoretical advantages of TriAL*

know expressive power: First Order logic with transitive closure and fixed amount of variables

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Can I pose this query?

Outline

- 1. Graph databases Most studied connectivity queries
- 2. Relationship between connectivity and recursion An algebra for querying graphs

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3. Where to go from here

What now?

Lets get back to what we know, and study connectivity queries from a relational perspective

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- Shortest path
- Count number of paths
- Aggregate on paths (total distance, etc)

What now?

Include TriAL in graph implementations

- Include it in SPARQL
- compare performance with Neo4j, Dex

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RDF DBMS (Jenna, etc)