Classification of Annotation Semirings over Query Containment

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Relational Database annotation

Relational Database annotation: *Comments*

Takes	*Student*	*Course*		
	Jane	Algebra	Top mark	Wants TA
	Jane	Physics		
	Anne	History	Class Rep.	

Likes	*Student*	*Course*	
	Jane	Algebra	Wants TA
	Anne	Literature	

Relational Database annotation: *Comments*

Takes	*Student*	*Course*		
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Likes	*Student*	*Course*	
	Jane	Algebra	Wants TA
	Anne	Literature	

SELECT	Student, Course
FROM	Takes, Likes
WHERE	Takes.S = Likes.S
AND	Takes.C = Likes.C

(Jane, Algebra):
Top mark Wants TA

Relational Database annotation: *Belief*

Takes	*Student*	*Course*		
	Jane	Algebra	Teach. Office	Stud. Union
	Jane	Physics		
	Anne	History	Teach. Office	

Likes	*Student*	*Course*	
	Jane	Algebra	Stud. Union
	Anne	Literature	

SELECT	Student, Course
FROM	Takes, Likes
WHERE	Takes.S = Likes.S
AND	Takes.C = Likes.C

(Jane, Algebra):
Stud. Union

Relational Database annotation: *Bag Semantics*

Takes	*Student*	*Course*	
	Jane	Algebra	2
	Jane	Physics	1
	Anne	History	3

Likes	*Student*	*Course*	
	Jane	Algebra	2
	Anne	Literature	1

SELECT	Student, Course
FROM	Takes, Likes
WHERE	Takes.S = Likes.S
AND	Takes.C = Likes.C

(Jane, Algebra):

$2 \times 2 = 4$

Relational Database annotation: *Fuzzy Databases*

Takes	*Student*	*Course*	
	Jane	Algebra	0.6
	Jane	Physics	0.3
	Anne	History	1

Likes	*Student*	*Course*	
	Jane	Algebra	0.5
	Anne	Literature	1

SELECT	Student, Course
FROM	Takes, Likes
WHERE	Takes.S = Likes.S
AND	Takes.C = Likes.C

(Jane, Algebra):
$0.6 \times 0.5 = 0.3$

Semirings

(Green et al. 07):

- Domains of annotations are commutative semirings.
- Typical example: natural numbers
- $\mathcal{K} = \langle K, +, \times, 0, 1 \rangle$

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More examples:

- *Comments*: $\langle \{c_1, c_2, c_3, \cdots\}, \cup, \uplus, \emptyset, \mathbb{U} \rangle$
- *Belief*: $\langle x, y, z, \ldots, \cup, \cap, \emptyset, \mathbb{U} \rangle$
- *Fuzzy* Databases $\langle [0, 1], \max, \times, 0, 1 \rangle$

Semirings

(Green et al. 07):

- Domains of annotations are commutative semirings.
- Typical example: natural numbers
- $\mathcal{K} = \langle K, +, \times, 0, 1 \rangle$

For query evaluation (positive relational algebra):

- Joins we Multiply the annotations
- Unions we Add the annotations

Query Containment

- Optimization
- Querying using views
- Information integration
- ...

We study query containment in annotated databases

What is so special about containment?

- Not the same as Set Semantics
- Varies depending on the annotation domain
- Open Problems (Bag Semantics)

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$$Q_1 := \exists u \exists v, \exists w \ \mathit{Takes}(u, v), \mathit{Takes}(u, w)$$
$$Q_2 := \exists u \exists v \ \mathit{Takes}(u, v)$$

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\end{aligned}
$$

Q_1 is contained in Q_2 under Set Semantics

Q_1 is not contained in Q_2 under Bag Semantics

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\end{aligned}
$$

Q_2 is contained in Q_1 under Set Semantics or Bag Semantics

Q_2 is not contained in Q_1 over fuzzy databases

Previous Work has focused on particular semirings

- Bag Semantics
- Probabilistic Databases
- Various semirings for provenance

But new applications may use new semirings

We focus on classes of semirings

Contributions

- Identify several classes of semirings for annotation with decision procedures for checking: containment of CQs and UCQs.
- Generalize previous work
- Some results by known techniques (homomorphisms)
- Others using new machinery, based on
  - Relationships between queries and polynomials
  - Small model properties

Outline

- Formalization of $\mathcal{K}$-containment
- Some results in the paper

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Query Evaluation on annotated databases

Bag Semantics: $\langle \mathbb{N}, +, \times \rangle$

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I:

Takes	*Student*	*Course*	#
	J	A	2
	J	P	1

Query Evaluation on annotated databases

Bag Semantics: $\langle \mathbb{N}, +, \times \rangle$

- For each homomorphism h from Q to I:
  1. Compute the annotation of $h(Q)$
  2. Sum over all homomorphisms.

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$$h(Q) \quad := \quad Takes(J, A), Takes(J, P)$$

I:

Takes	Student	Course	#
	J	A	2
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$Q_1(I) = 2 \cdot 1$

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$$\begin{aligned} Q &:= \exists u, \exists v, \exists w \ \ Takes(u, v), Takes(u, w) \\ h(Q) &:= Takes(J, P), Takes(J, A) \end{aligned}$$

I:

Takes	Student	Course	#
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$Q_1(I) = 2 \cdot 1 + 1 \cdot 2$

Query Evaluation on annotated databases

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I:

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	J	A	2
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$$Q_1(I) = 2 \cdot 1 + 1 \cdot 2 + 2 \cdot 2 + 1 \cdot 1 = 9$$

Query Evaluation on annotated databases

Fuzzy Databases: $\langle [0,1], \max, \times \rangle$

- For each homomorphism h from Q to I:
  1. Compute the annotation of $h(Q)$
  - {2.] Sum over all homomorphisms.

$$Q \quad := \quad \exists u, \exists v, \exists w \;\; Takes(u,v), Takes(u,w)$$

I:

Takes	*Student*	*Course*	*Probability*
	J	A	0.7
	J	P	0.3

$$Q_1(I) = \max\left(0.7 \times 0.3, 0.3 \times 0.7, 0.7 \times 0.7, 0.3 \times 0.3\right) = 0.49$$

Query Containment over Annotated Databases

- Semirings with partial order $\preceq_{\mathcal{K}}$
- For Bag Semantics, Fuzzy databases we use the order $\leq$
- For comments, belief, provenance we use order $\subseteq$:
  $\{\text{Wants TA}\} \subseteq \{\text{Top Mark}, \text{Wants TA}\}$

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Definition of containment (boolean queries):

Q_1 is $\mathcal{K}$-contained in $Q_2 \Leftrightarrow Q_1(I) \preceq_{\mathcal{K}} Q_2(I)$, for all instances I

- Write $Q_1 \subseteq_{\mathcal{K}} Q_2$

Outline

- Formalization of $\mathcal{K}$-containment
- Some results in the paper

Previous Work

Set semantics: $\langle \{0, 1\}, \vee, \wedge \rangle$

$\mathcal{B}$

Previous Work - Chandra & Merlin '77

$Q_1 \subseteq Q_2$ iff
homomorphism from Q_2 to Q_1.

Set semantics: $\langle \{0, 1\}, \vee, \wedge \rangle$

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Previous Work

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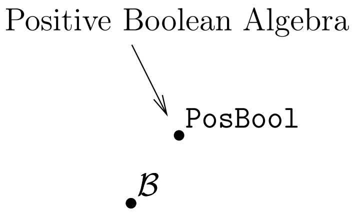
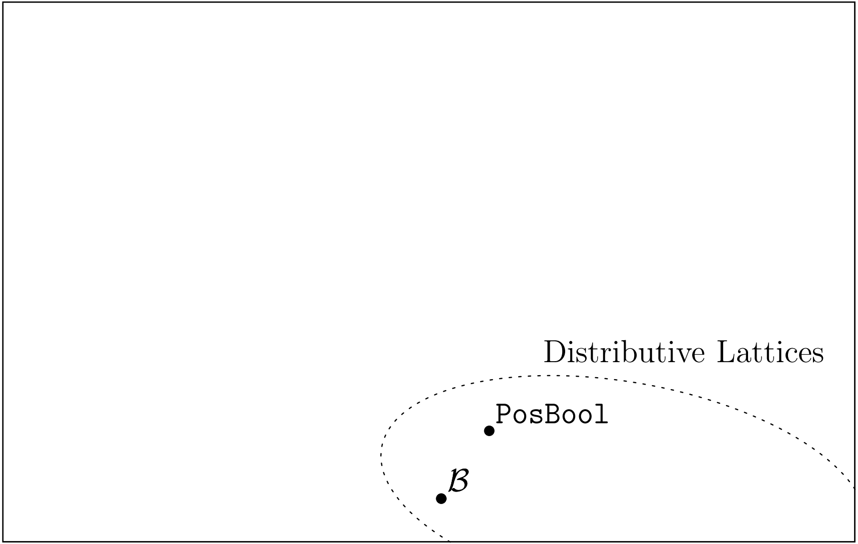


Previous Work - Grahne et al. '97



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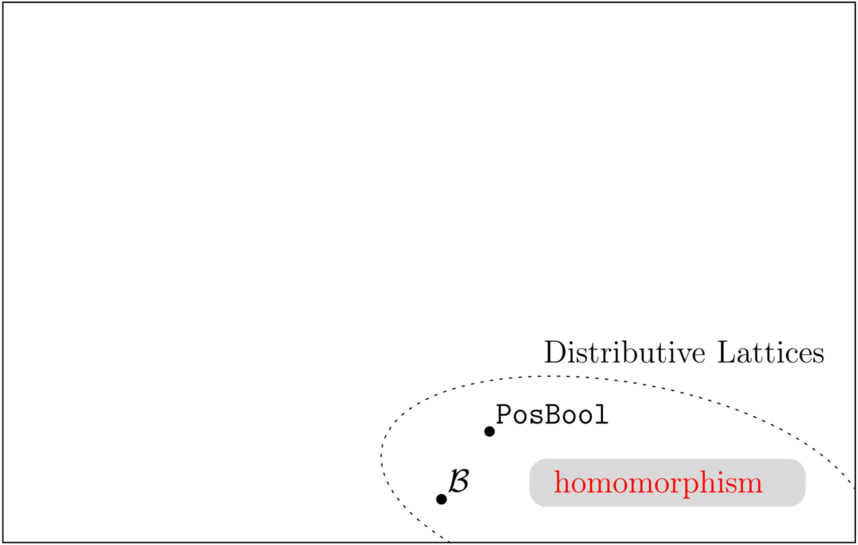

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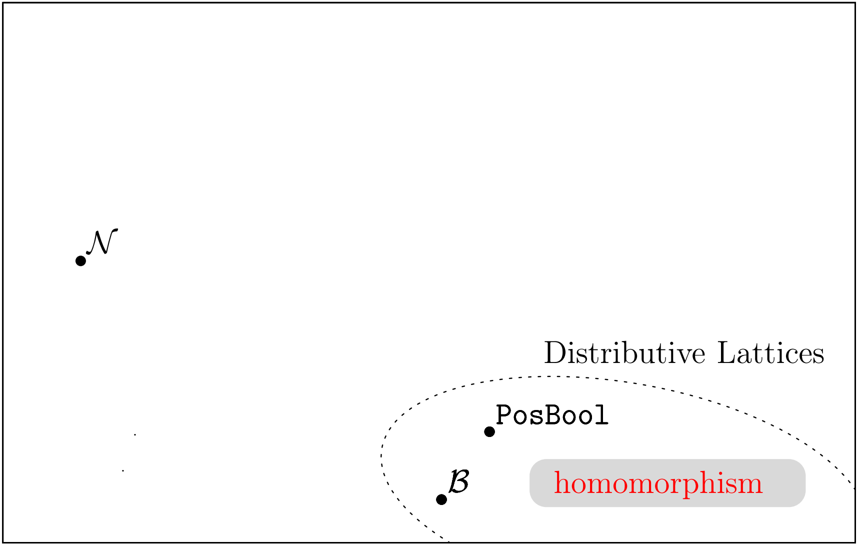
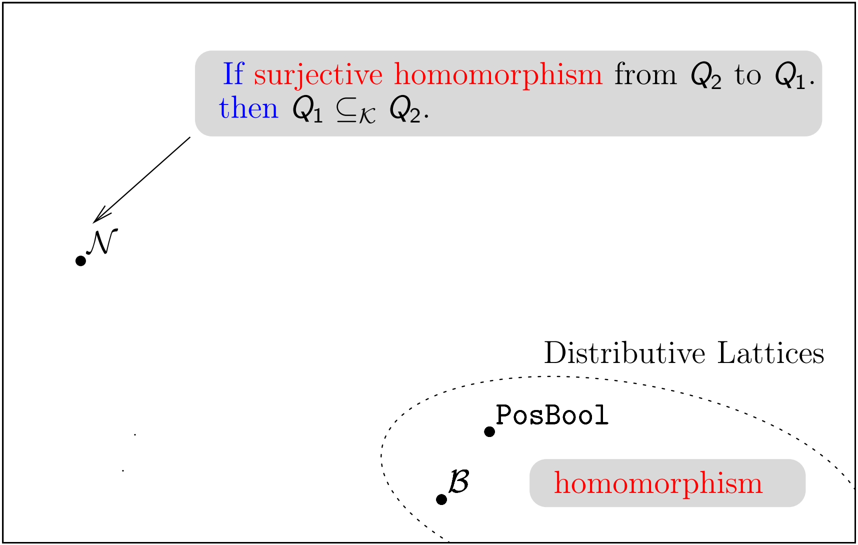


Previous Work - Chaudhuri & Vardi '93



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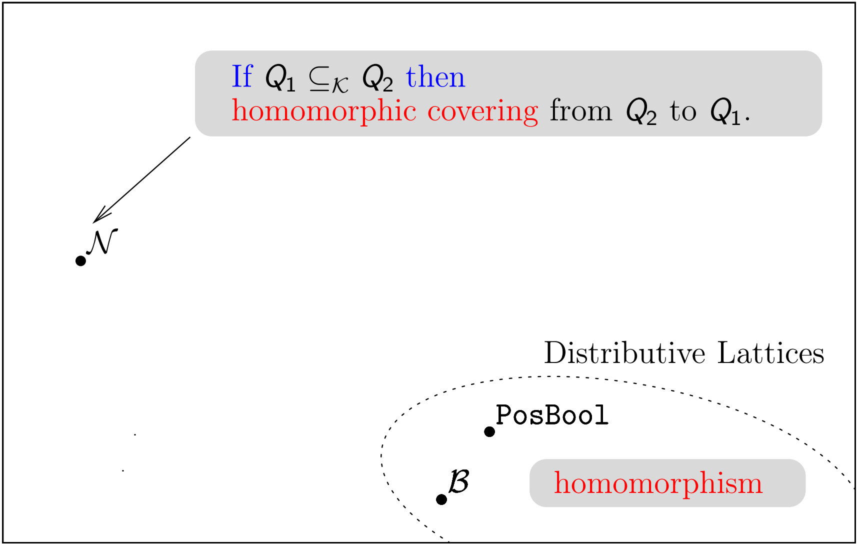
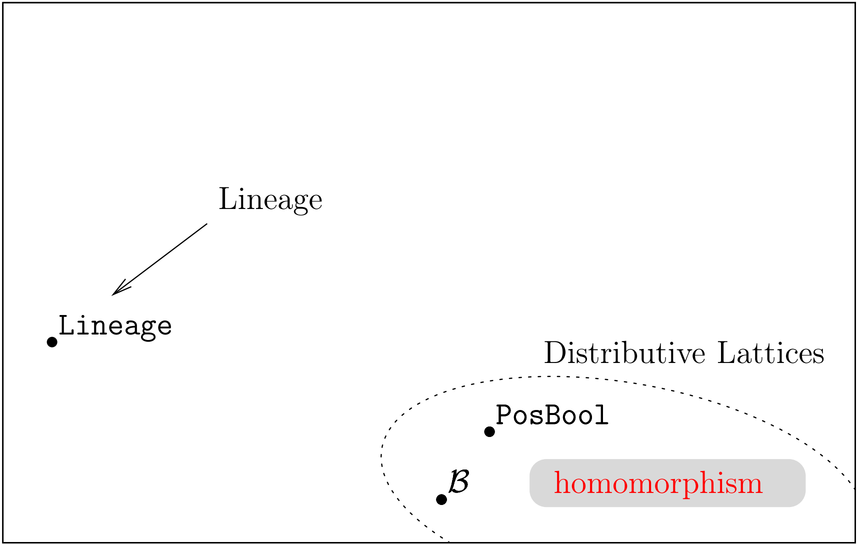


Previous Work - Green '09



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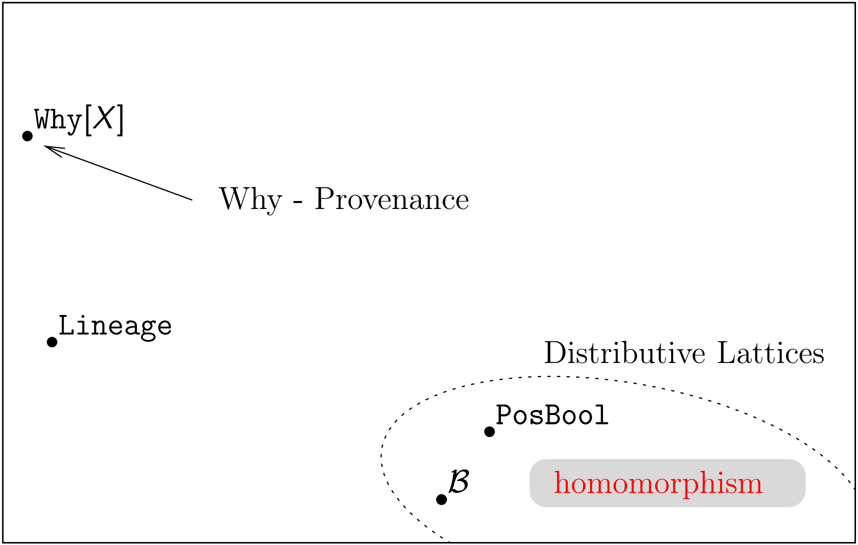

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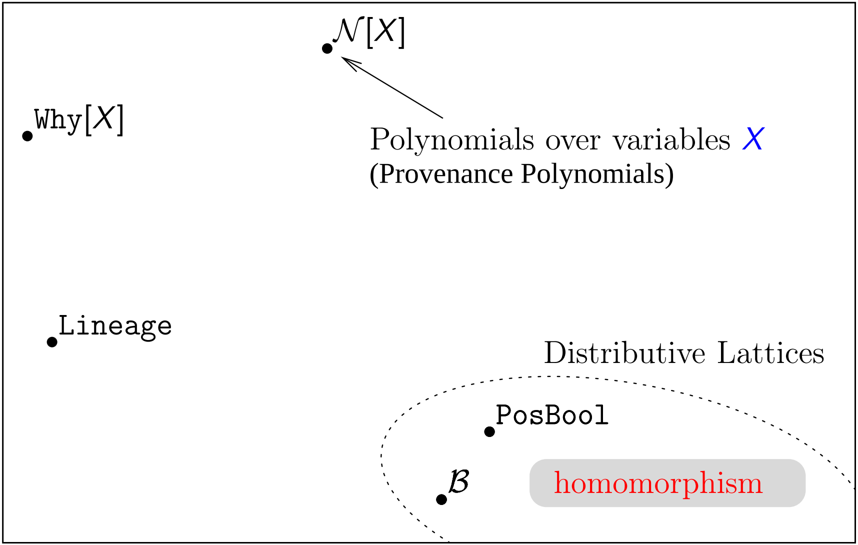

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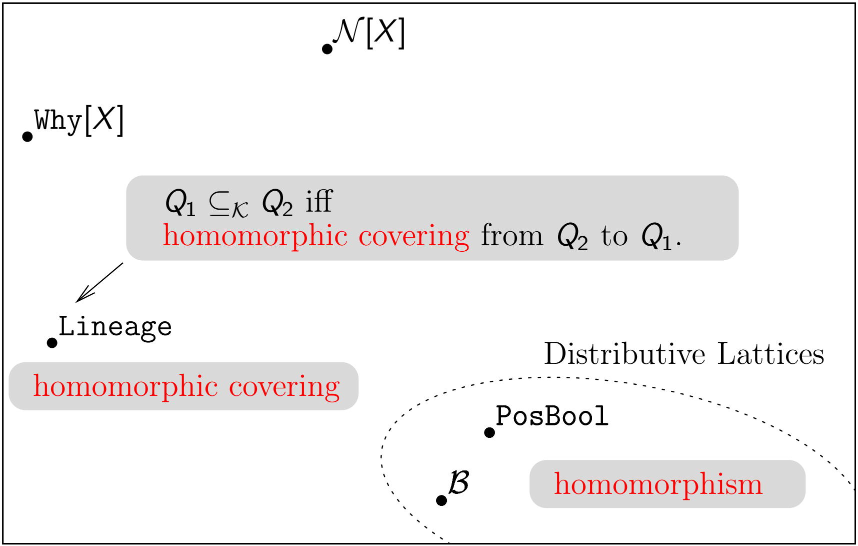

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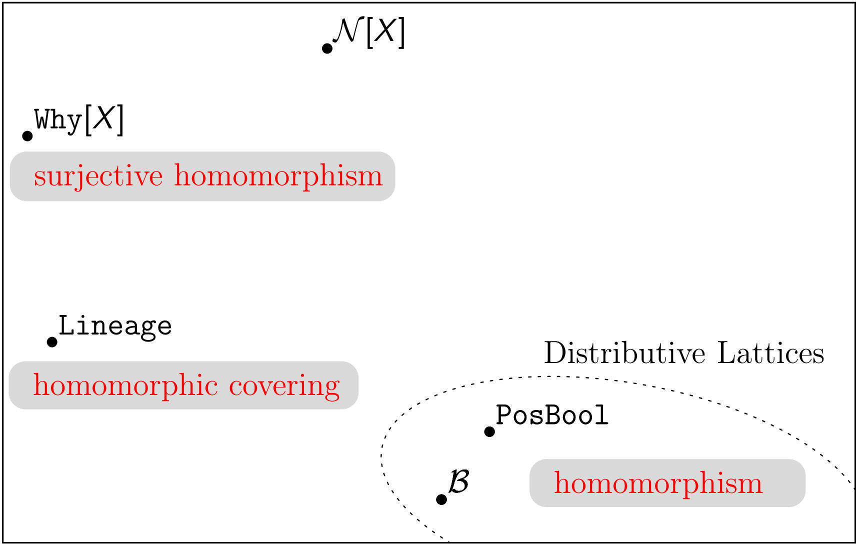

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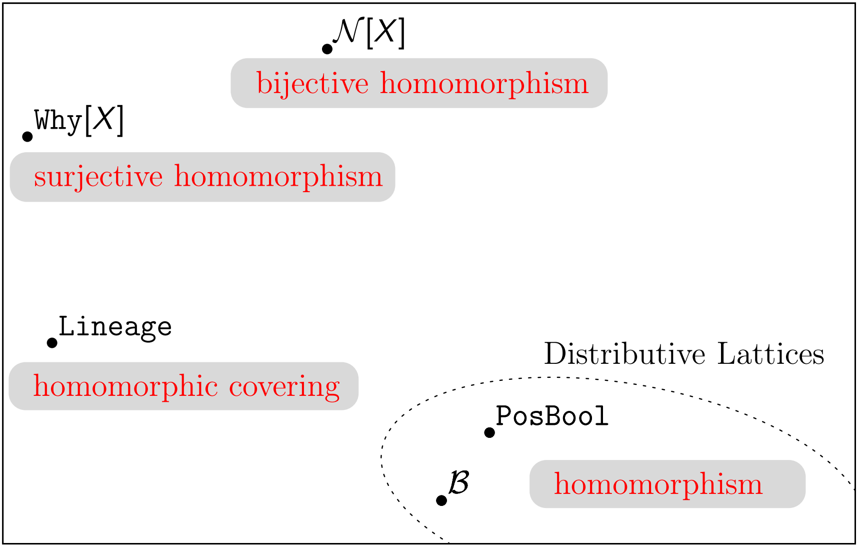

Summing up, we have:

- Different types of mappings (homomorphisms)
- For a semiring $\mathcal{K}$ they can be:
- *Sufficient condition* for containment
- *Necessary condition* for containment
- *Decision procedure* for containment

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If mapping from Q_2 to Q_1 then $Q_1 \subseteq_{\mathcal{K}} Q_2$

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If $Q_1 \subseteq_{\mathcal{K}} Q_2$ then mapping from Q_2 to Q_1

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Summing up, we have:

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$Q_1 \subseteq_{\mathcal{K}} Q_2$ iff mapping from Q_2 to Q_1

We fully characterize the universe of semirings

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- Axiomatize classes of semirings for which different type of mappings are sufficient, or necessary conditions for $\mathcal{K}$-containment of CQ's
- Several classes for which $\mathcal{K}$-containment is decidable

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- Several classes for which $\mathcal{K}$-containment is decidable
- Generalize to Unions of CQs
- Additional decision procedures for $\mathcal{K}$-containment

Outline

- Formalization of $\mathcal{K}$-containment
- Some results in the paper
- Results for *homomorphisms*
- Results for *homomorphic covering*...

  and a relevant class of *polynomials*

Containment of CQ's for set semantics

- Model set semantics as $\mathcal{B} = \langle \{0, 1\}, \vee, \wedge, 0, 1 \rangle$

Q_1 is $\mathcal{B}$-contained in Q_2 iff
there is a homomorphism from Q_2 to Q_1

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Is this true for any other semiring?

Many semirings behave as set semantics

- Boolean Algebra
- Event tables
- Type A systems (Ioannidis et al. 95)
- Distributive lattices

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Can we characterize all semirings with this behavior?

Yes we can

A semiring $\mathcal{K}$ is in $\mathcal{H}$ if

1. $a \times a = a$
2. $1 + a = 1$

for all $a \in \mathcal{K}$.

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Theorem

$\mathcal{H}$ captures precisely all semirings that behave as Set Semantics (wrt. containment of CQs)

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If $\mathcal{K}$ is in $\mathcal{H}$ then

- Homomorphism is a decision procedure for $\mathcal{K}$-containment

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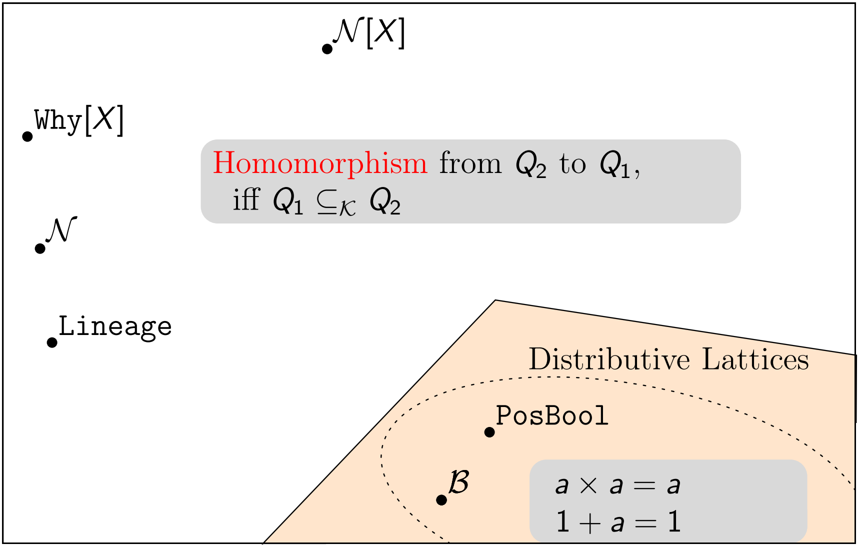
If Homomorphism is a decision procedure for $\mathcal{K}$-containment

- Then $\mathcal{K}$ is in $\mathcal{H}$

Class $\mathcal{H}$



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Moving away from $\mathcal{H}$

Two options:

- Keep $a \times a = a$
- Keep $1 + a = 1$

Moving away from $\mathcal{H}$

Two options:

- Keep $a \times a = a$
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Example:

- Lineage $\texttt{Lineage} = \langle \{x, y, z, w, \dots\}, \cup, \uplus \rangle$

Semirings satisfying $a \times a = a$

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- Homomorphisms are not sufficient condition

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\begin{aligned}
Q_1 &:= \exists u, \exists v, \exists w \;\; Takes(u, v), Likes(u, w) \\
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\end{aligned}
$$

- Homomorphism from Q_2 to Q_1
- Q_1 is not `Lineage-contained` in Q_2

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I:

Takes	*Student*	*Course*	*Lineage*
	J	A	x
	J	P	x

Likes	*Student*	*Course*	*Lineage*
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Likes	*Student*	*Course*	*Lineage*
	J	A	y

- $Q_1(I) = \{x, y\}$
- $Q_2(I) = \{x\}$

We need a stricter notion of mapping

Idea:

- force both queries to target the same relations

Homomorphic Covering from Q_1 to Q_2

Intuition:
Cover each atom of Q_2 with a homomorphism from Q_1 to Q_2

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There is a homomorphic covering from Q_1 to Q_2

$$
\begin{aligned}
Q_3 &:= \exists u, \exists v \;\; Takes(u, v) \\
Q_4 &:= \exists u, \exists v, \exists w \;\; Takes(u, v), Likes(u, w)
\end{aligned}
$$

There is no homomorphic covering from Q_3 to Q_4

We can now capture semirings satisfying $a \times a = a$

Let $\mathcal{K}$ be a semiring.

Theorem

If $\mathcal{K}$ satisfies $a \times a = a$

- *Then Homomorphic covering is a sufficient condition for $\mathcal{K}$-containment*

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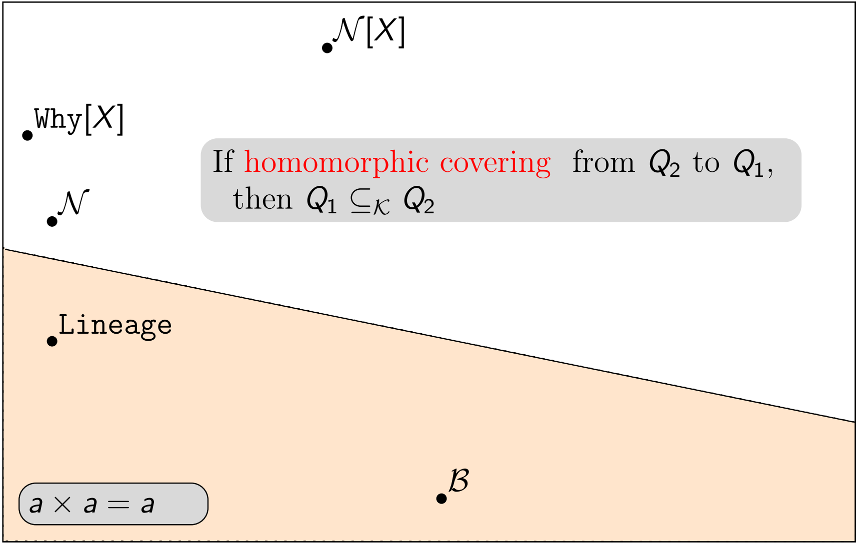
$a \times a = a$ captures homomorphic covering, as sufficient condition.

Class $\mathcal{H}$



Homomorphic covering as necessary condition

semirings $\mathcal{K}$ for which homomorphic covering is a necessary condition for $\mathcal{K}$-containment?

- Bag Semantics $\mathbb{N}$ should belong to this class.
- We axiomatize this class
- By abstracting query evaluation into polynomials.

CQ-admissible polynomials

When annotating each tuple with a different variable:

Evaluation of queries correspond to polynomials

- We need to understand the *structure* of these polynomials

$$Q_1 \quad := \quad \exists u, \exists v, \exists w \;\; Takes(u, v), Takes(u, w)$$

I:

Takes	Student	Course	P
	J	A	x
	J	P	y

$$Q_1(I) = x \cdot y + y \cdot x + x \cdot x + y \cdot y = x^2 + 2xy + y^2$$

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Obtained from evaluating a CQ over an instance annotated with (different) variables.

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- Only homogeneous polynomials
- Precise definition is more technical

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In the paper:

Syntactic characterization of CQ-admissible polynomials.

Homomorphic covering as necessary condition

Using CQ-admissible polynomials
we define a class $\mathcal{C}$ of semirings, such that:

Theorem

$\mathcal{C}$ captures homomorphic coverings as a necessary condition

Note that $\mathcal{C}$ contains Bag Semantics, but not Set Semantics.

And obtain a class where containment is decidable

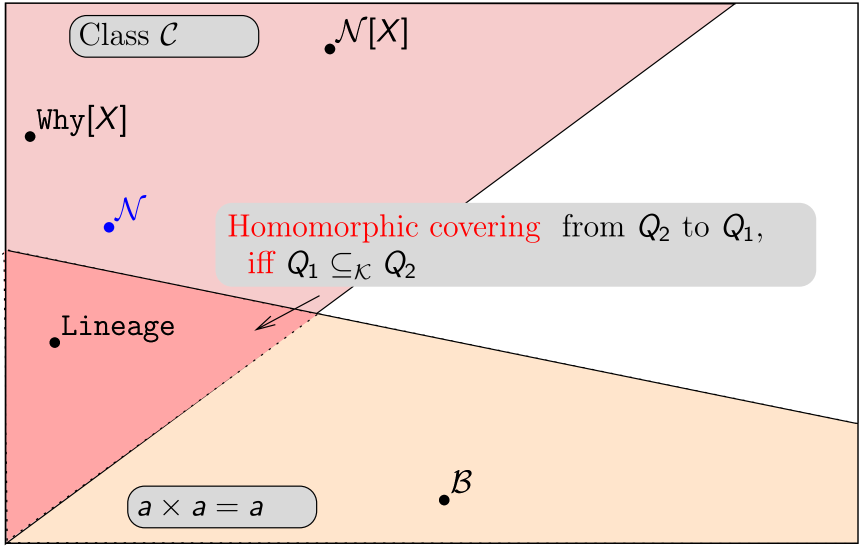
The following are equivalent:

Theorem

- $\mathcal{K}$ *belongs to* $\mathcal{C}$ *and satisfies* $a \times a = a$
- $Q_1 \subseteq_{\mathcal{K}} Q_2$ *iff homomorphic covering from* Q_2 *to* Q_1

Gives us a large *class of semirings* where $\mathcal{K}$-containment is decidable

And obtain a class where containment is decidable



Also in the paper

- Similar theorems for surjective homomorphism, injective homomorphisms and bijective homomorphisms
- Extension to UCQs
- Complete Descriptions of CQs and UCQs
- Small model property and new procedures for semirings satisfying

$$a + a = a$$

Future work

- Well behaved Semirings:

$$a + a = a$$

- Containment of CQ-admissible polynomials over various semirings
- Views over annotated databases