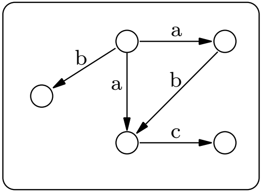
Querying Graph Patterns

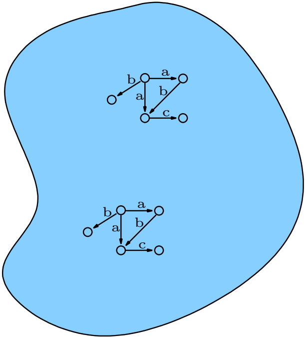
Pablo Barceló	Leonid Libkin	Juan Reutter
U. de Chile	U. of Edinburgh	U. of Edinburgh

Graph Patterns

Graph Pattern



Labeled Graph G



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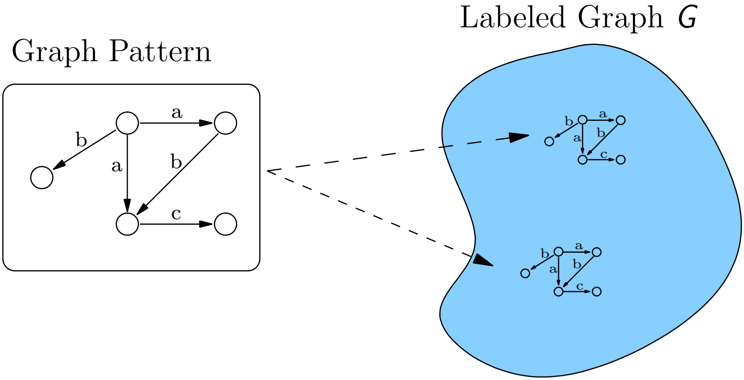

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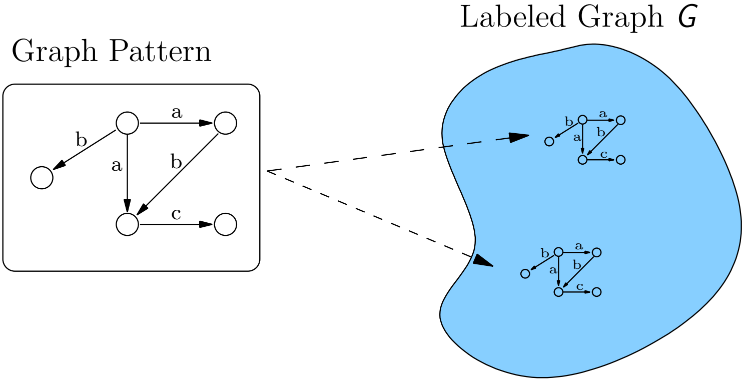
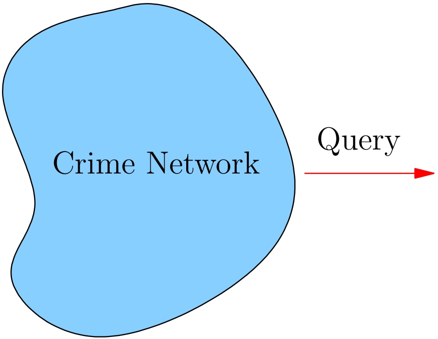


Applications in biology, social networks,
semantic web, and many others.

We often end up querying patterns rather than graphs



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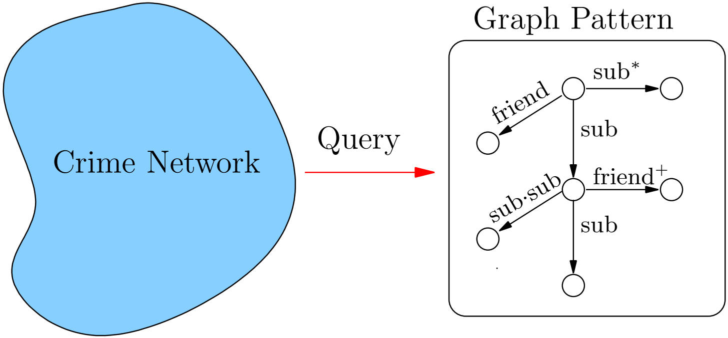

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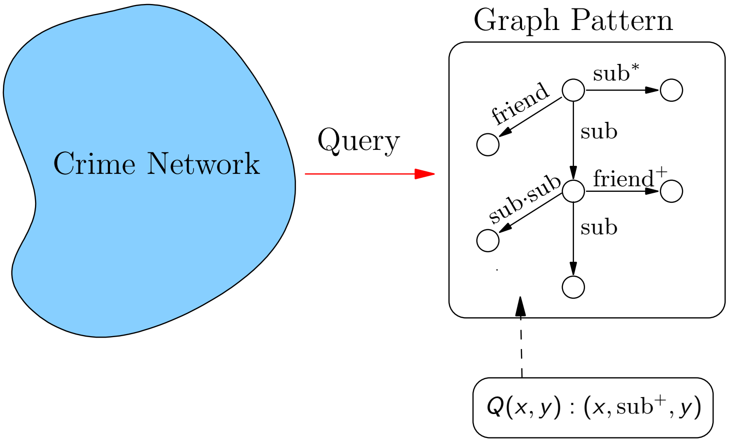

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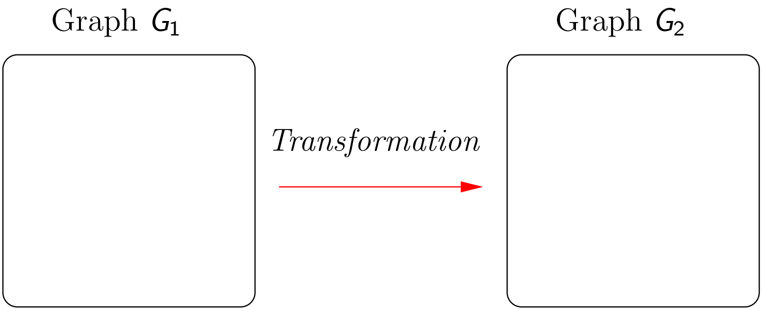

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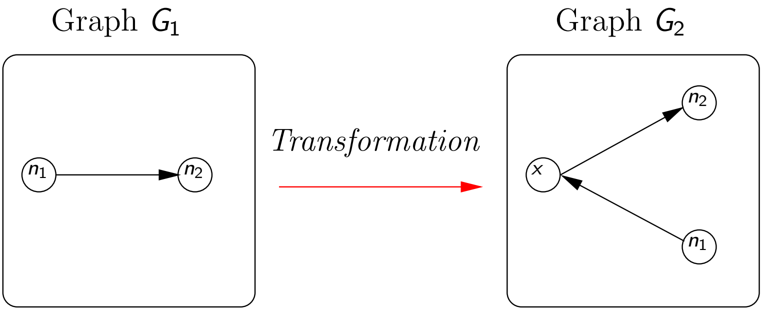

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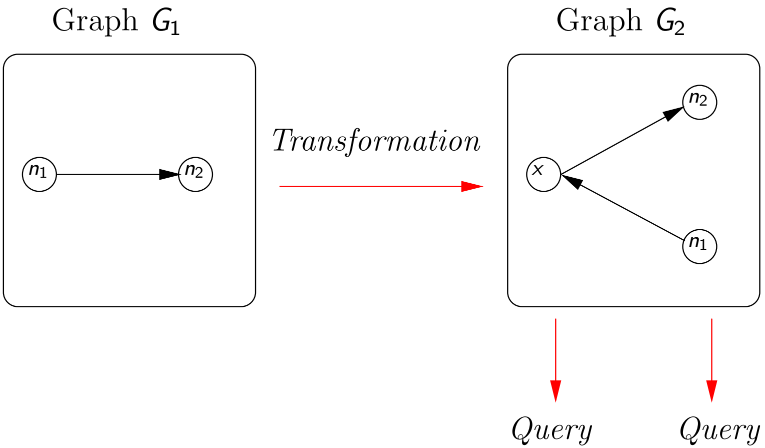

We study how to query partially defined graph data

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- This data is represented by means of patterns, with some additional features.

Representing partially defined data: Relations

S

D	E
4	5
z	8
y	y

R

A	B	C
1	3	x
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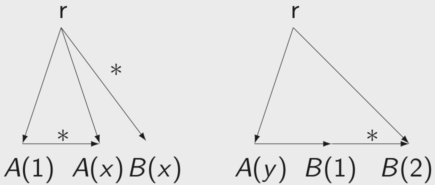

Representing partially defined data: Graph DB's

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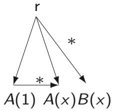
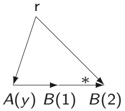


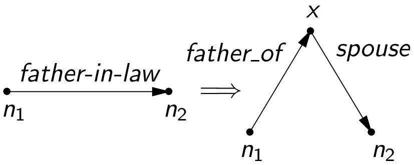


Next: features that need to be addressed in the study of querying graph patterns.

- Examples from social networks

Key features of graph patterns

For graph patterns, incomplete specification may arise in 3 ways:



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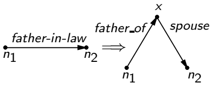

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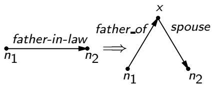

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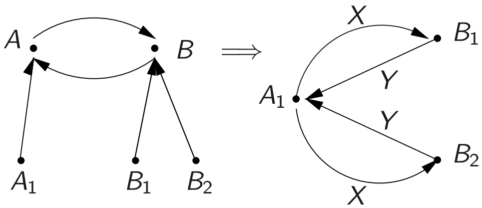

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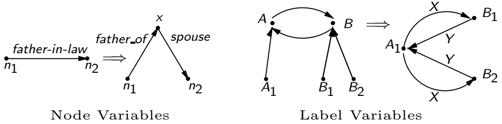

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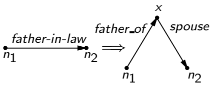

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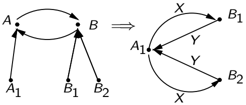

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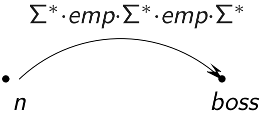

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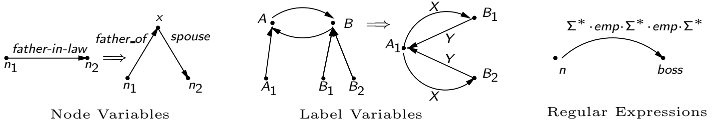

For graph patterns, incomplete specification may arise in 3 ways:

- Node Variables
- Label Variables
- Regular Expressions

We study how to query partially defined graph data

Outline

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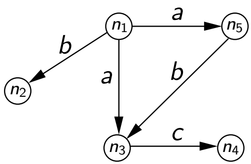
Graph Databases



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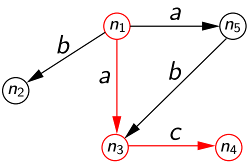

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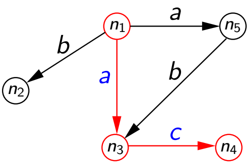
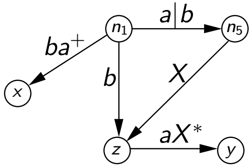
Graph Databases



- Path from n_1 to n_4
- The *label* of the path is ac

Graph patterns



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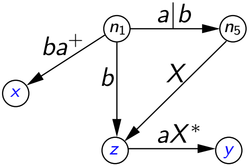

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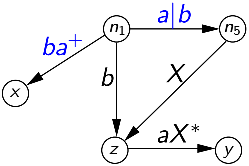

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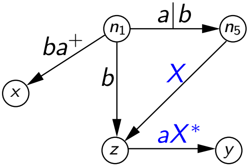

- Node Variables
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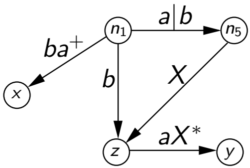
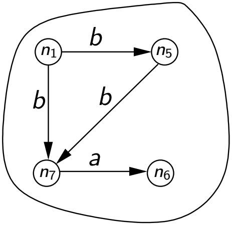
Semantics of graph patterns: by homomorphisms





Graph homomorphism maps

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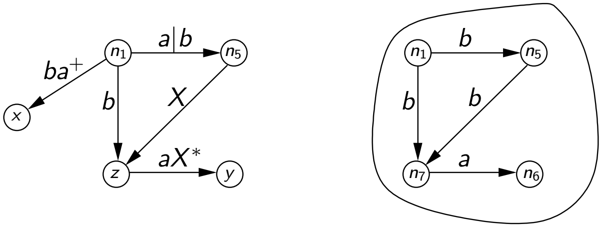

Graph homomorphism maps

- Nodes to Nodes
- Edge label variables to Σ
- Edge expressions are witnessed by paths

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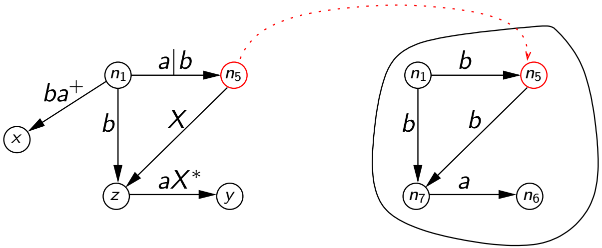

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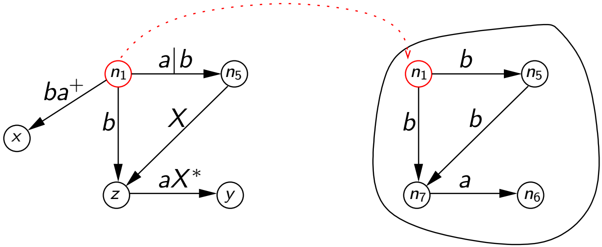

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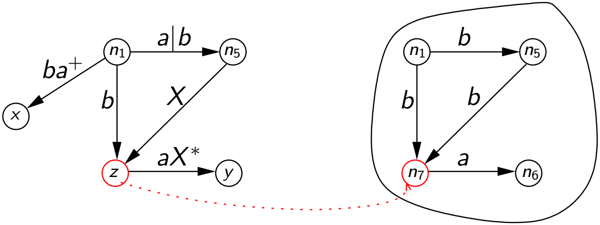

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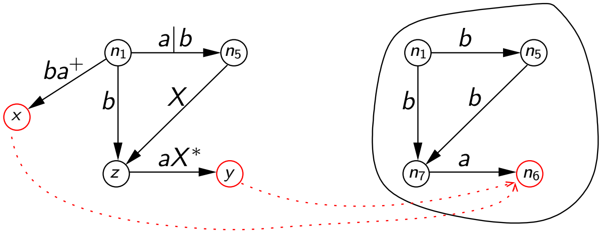

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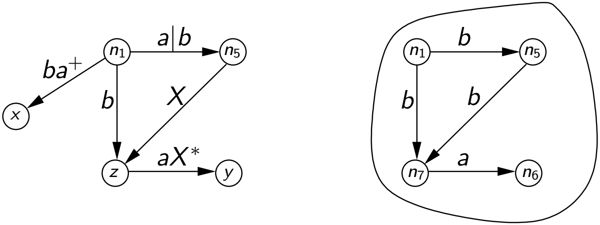

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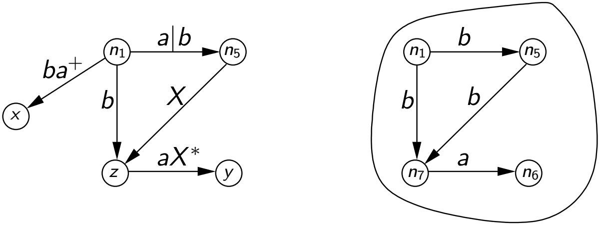

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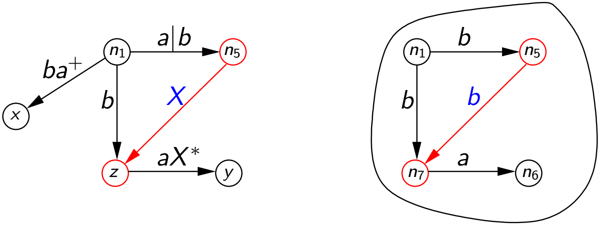

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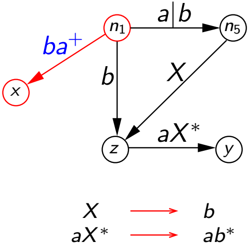

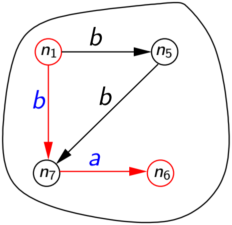

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Each pattern π represents an infinite set $[\![\pi]\!]$ of graph databases:

$$[\![\pi]\!] = \{G \mid \text{there is a homomorphism from } \pi \text{ to } G\}$$

Using Graph Patterns to query databases

A graph query Q consists of:

- a graph pattern
- a tuple of output variables

Intuitively,

Select all tuples of nodes that *realize* the pattern on a graph.

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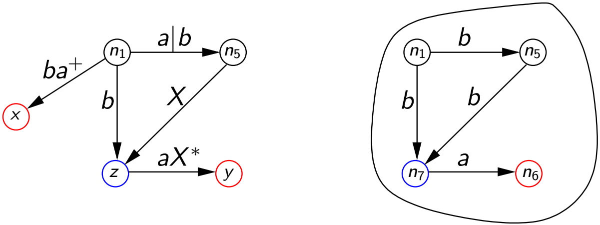

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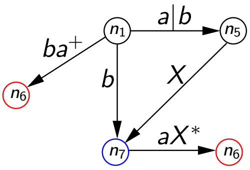

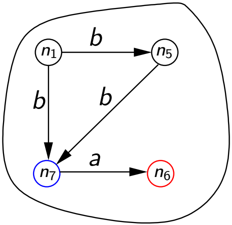

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Defining Classes of patterns according to their features

Features:	node variables (nv) label variables (lv) regular expressions in the edges (re)

We define $\mathcal{P}^{\sigma}$, for $\sigma \subseteq \{\mathrm{nv}, \mathrm{lv}, re\}$

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- $\mathcal{P}$: subgraph isomorphism
- $\mathcal{P}^{\mathrm{nv},\mathit{re}}$: Essentially CRPQ queries
- $\mathcal{P}^{\mathrm{nv},\mathrm{lv}}$: Can only use variables, but no expression in the edges

We classify classes of patterns in terms of the sets that they can represent

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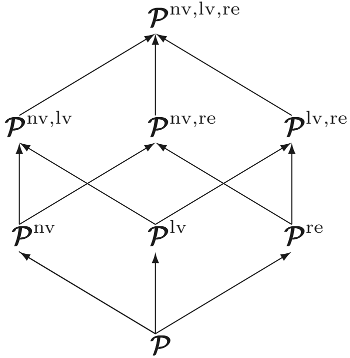

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By means of *certain answers*:

$$\mathrm{certain}(Q, \pi) \; = \; \bigcap \{ Q(G) \mid G \in [\![\pi]\!] \}.$$

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By means of *certain answers*:

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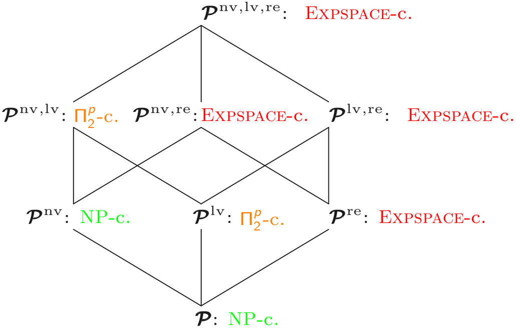
- Combined and data complexity
- Full classification for CRPQs
- Upper bounds are maintained for the *most general case*

Combined complexity

For CRPQ's:



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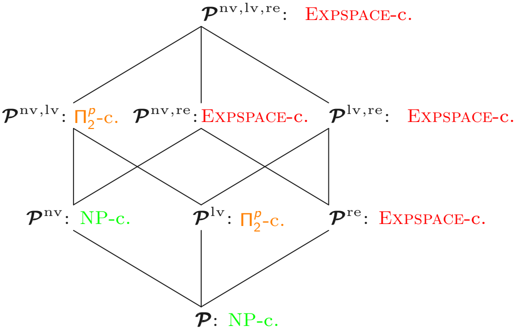
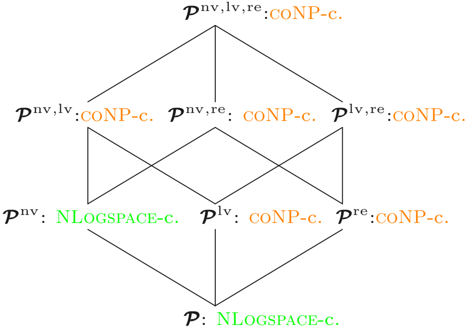


General upper bound:

The combined complexity of arbitrary graph queries on arbitrary patterns is in EXPSPACE.

Data Complexity

For CRPQ's:



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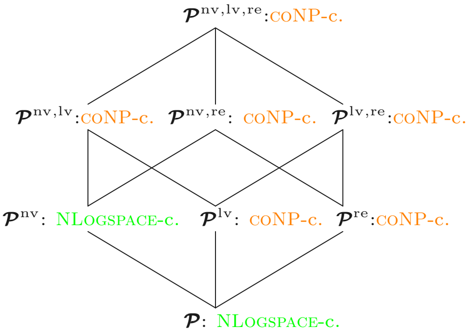

General upper bound:

Data complexity of arbitrary graph queries over arbitrary graph patterns is in coNP.

Outline

Finding Tractable Clases (data complexity)

- $\mathcal{P}^{\mathrm{nv}}$: naive tables

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- Intractable for $\mathcal{P}^{\mathrm{lv}}$ or $\mathcal{P}^{\mathrm{re}}$

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Restrictions:

- *Structure:* underlying graphs
- Codd patterns

A pattern is in $\mathcal{P}^{\mathrm{nv,lv}}_{\mathrm{Codd}}$ if every variable occurs at most once in it

Finding Tractable Clases (data complexity)

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coNP-hard for *paths* in $\mathcal{P}^{\mathrm{lv}}$.

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Finding Tractable Clases (data complexity)

Attempts to put restriction in the structure are not very good:

coNP-hard for *paths* in $\mathcal{P}^{\mathrm{lv}}$.

coNP-hard for *DAGs* in $\mathcal{P}^{\mathrm{re}}$ or *DAGs* in $\mathcal{P}^{\mathrm{lv}}_{\mathrm{Codd}}$.

- The only possibility appears to be patterns in $\mathcal{P}^{\mathrm{nv,lv}}_{\mathrm{Codd}}$ or $\mathcal{P}^{\mathrm{nv,re}}$ with *nice underlying graphs*.

Bounded treewidth gives us tractability (data complexity)

- The treewidth of a pattern π is the treewitdth of its underlying graph.

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Theorem:

Query answering is in $\textsc{Ptime}$ for CRPQ's over classes of patterns in $\mathcal{P}^{\text{nv,lv}}_{\text{Codd}}$ or $\mathcal{P}^{\text{nv,re}}$ with bounded treewitdh.

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Certain paths:

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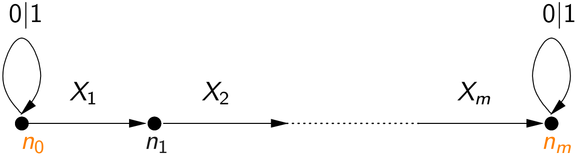

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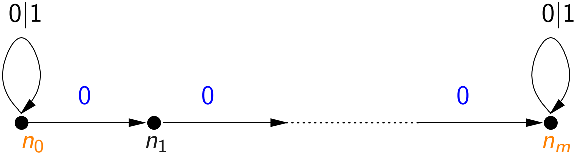

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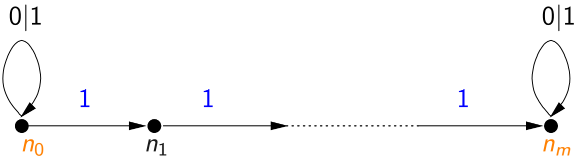

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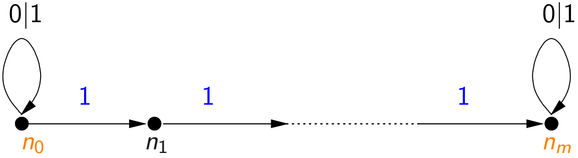

- Certain paths contain all words of length m
- The size of each certain path is *greater* than 2^m

We introduce Incomplete automata

In essence, a graph pattern with distinguished initial and final nodes

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Some results:

- The *smallest* NFA representing the certain paths can be doubly-exponential in the size of π.
- Checking wether there exists a certain path is EXPSPACE-complete.
- Checking if a word is a certain path is coNP-complete.

We study how to query graph patterns

How *features* of patterns affect query answering:

- Each feature strictly increases the expressivity of patterns
- Features have to be carefully chosen to guarantee tractability
- We identify classes with PTIME query answering
- And others for which we hope to find good heuristics

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Starting point for research in *schema mappings*, *exchanging* and integrating graph data.

Querying Graph Patterns

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